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Self-avoiding path walks on lattices—a new universality class?

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Abstract. The trails problem (self-avoiding path walks) is reconsidered by the conventional series expansion approach. An exact enumeration on the square, triangular and simple cubic lattices is done by computer up to 18, 12 and 11 steps respectively. The more self-consistent and refined results, between the connective constant and the corresponding critical exponent, indicate the possibility of there being a different universality class from that of the self-avoiding walk. An average approach based on Stolz's theorem is proposed which often appears to be smoother and better than the traditional one.

1. Introduction

The random walks with various simple and peculiar global excluded volume effects, i.e. some kinds of long-range correlation, have been actively studied for some years (for reviews, see e.g. Essam 1980, Domb 1963, de Gennes 1979, McKenzie 1976). The self-avoiding walk (SAW) is the most prominent example, for it provides a simple model for polymer chains which takes into account in a realistic manner the excluded volume effect (Domb 1963). It is unthinkable that one can propose other models simpler than SAWs to consider the excluded volume effect, which is perhaps the most important physical effect for polymers. Unfortunately, one can only use various numerical methods but not analytic ones to approach this elegant simple model because of its formidable mathematical complexities.

Another model of random walks, still with some simple and peculiar global excluded volume effects, the 'trails' problem, is seldom studied (Malakis 1975, 1976). In a trail no (lattice) edge occurs (or is visited) more than once. Here we prefer to call it a self-avoiding path walk (SAPW) because of the direct and clear meaning presented in this terminology. An asymptotic analysis of the previous data on the square lattice strongly suggested that certain critical exponents obey the same values for both the SAPW and SAW (Malakis 1975, 1976). However, as pointed out by the same author, there is no simple relation between SAPWs and SAWs on its covering lattice. The most one can say is: 'to every trail on a lattice there corresponds a SAW on its covering lattice but not *vice versa*'. In other words, only a homomorphic but not an isomorphic relation exists between SAPWs and SAWs on the covering lattice, and it is not possible from this relation to judge whether SAPWs and SAWs belong to the same universality class.

A direct and a cell-to-cell renormalisation approach for the SAPW on the square lattice were done by us recently (Li *et al* 1984). But one cannot give a conclusive result as far as the universality class is concerned by use of the small cell one-parameter PSRG approach.

Since the SAPW has a very different global excluded volume effect from the SAW, the difference cannot be eliminated by any scaling transformation, as happens for short-range correlations. One has reason to expect the possibility of a new universality class for the SAPW. Obviously, it is an interesting and worthy problem. We checked the previous series expansion results (Malakis 1976), and found that some errors may happen in their exact enumeration, which is the fundamental starting point for the series expansion methods. Also some unsatisfactory inconsistencies exist in their numerical results. We will give some details in the following sections. Of course through Malakis's work some useful information is provided. In particular, after comparing with the parallel data of SAWs, he found that the behaviours of the various estimates appear to be smoother for the SAPW problem than for the SAW problem. This gives us some confidence to refine the results by the series expansion approach for SAPWS.

An exact enumeration on the square, triangular and simple cubic lattices was done by computer up to 18, 12 and 11 steps respectively. Our results for the SAPW indicate that it may belong to a new universality class, in contrast with the previous strong suggestion of the same class as the SAW.

2. Connective constants and the susceptibility exponents

The starting point for series expansion is the exact enumeration for the number C_N of N-step SAPWs on a lattice (Domb 1963). Firstly, one has the symmetry of a given lattice. Thus one only needs to enumerate the C_N paths with their first step on one of the Z nearest-neighbour sites, i.e.

$$C_N/Z = \tilde{C}_N. \tag{1}$$

Secondly, one can use further the symmetry of a given lattice according to the elegant enumeration method (Martin 1974). For example, for $\tilde{C}_N$ on the square lattice, for any path except the straight line (along, say, the right direction), it can go up or down after $m(<N)$ single direction steps. Since these two categories of paths have an isomorphic relation, thus one has

$$\tilde{C}_{N,\mathrm{s}} = 2m + 1, \tag{2}$$

$$\tilde{C}_{N,\mathrm{t}} = 2m + 1, \tag{3}$$

$$\tilde{C}_{N,\mathrm{sc}} = 4m + 1, \tag{4}$$

where the subscripts s, t and sc mean square, triangular and simple cubic lattice respectively. It is obvious that (1)–(4) are valid both for SAWs and SAPWs. However, one can find that (2) is not fulfilled for some C'_Ns ($N \geq 11$) in a previous paper (Malakis 1976, table 1), and we do not think that they are misprints, for our C'_Ns deviate considerably from theirs when $N \geq 11$. Thus it is worth redoing even a conventional series expansion for the SAPW.

We give the total number of N-step SAPWs on the square, triangular and simple cubic lattices in table 1. As for RWs and SAWs, we define the exponents γ and ν by the following asymptotic relations for large N for the SAPW:

$$C_N \sim N^{\gamma-1}\mu^N, \tag{5}$$

$$\rho_N \sim AN^{2\nu}, \tag{6}$$

Table 1. The SAPW problem on the square (SQ), triangular (TR) and simple cubic (SC) lattices. C_N and ρ_N are the number and the mean square end-to-end distance of n-step SAPWs respectively.

	SQ lattice		TR lattice	SC lattice	
N	C_N	$C_N\rho_N$	C_N	C_N	$C_N\rho_N$
2	12	32	30	30	72
3	36	164	150	150	582
4	108	704	738	750	4 032
5	316	2 748	3 570	3 726	25 758
6	916	10 096	17 118	18 438	156 504
7	2 628	35 524	81 498	90 966	918 390
8	7 500	121 056	385 710	447 918	5 254 368
9	21 268	402 420	1 817 046	2 201 622	29 482 998
10	60 092	1 311 504	8 528 478	10 809 006	162 926 040
11	169 092	4 205 480	39 903 462	52 999 446	889 246 854
12	474 924	13 304 860	186 198 642		
13	1 329 188	41 612 328			
14	3 715 244	128 878 688			
15	10 359 636	395 767 164			
16	28 856 252	1 206 315 216			
17	80 220 244	3 652 737 976			
18	222 847 804	10 995 975 680			

where ρ_N is the mean square end-to-end distance of an N-step SAPW. We then define

$$\mu_N \equiv C_N / C_{N-1} \tag{7}$$

which is different from Malakis (1976), but identical with that appearing in the current literature (e.g. Domb 1963). In table 2 we give these corresponding successive ratios for three lattices. The linear projections and their mean values are defined as follows:

$$X(n, m; \varepsilon) = (n-m)^{-1}[(n+\varepsilon)X_n - (m+\varepsilon)X_m], \qquad 0 \leqslant \varepsilon \leqslant \tfrac{1}{2}, \tag{8}$$

$$\bar{X}(n, m; \varepsilon) = \tfrac{1}{2}[X(n, m; \varepsilon) + X(n-1, m-1; \varepsilon)], \qquad 0 \leqslant \varepsilon \leqslant \tfrac{1}{2}. \tag{9}$$

From (8) and (9), one has $\bar{X}(n, n-1; \varepsilon) = X(n, n-2; \varepsilon)$, and their values are listed in table 2. For the loose-packed square lattice and simple cubic lattice, we took $m = n-2$ to divide the different properties for even and odd numbers of N; also the corresponding average values are listed in table 2. From table 2 we deduce the following estimates of connective constants:

$$\mu_s = 2.718 \pm 0.002 \approx \mathrm{e}, \tag{10}$$

$$\mu_t = 4.525 \pm 0.006, \tag{11}$$

$$\mu_{sc} = 4.850 \pm 0.001. \tag{12}$$

These estimates of μs are very consistent with the extrapolated values obtained from the nearly straight lines in the plots of μ_N against $1/N$ (figure 1). From these estimates of μs we get the corresponding approximate susceptibility exponents γ_Ns, according to the following formula:

$$\gamma'_n - 1 = N(\mu_N/\mu' - 1) \sim (\gamma - 1)(1 + \mathrm{O}(1/N)). \tag{13}$$

In table 2, some values of connective constants around the estimates in (10)–(12) are

Table 2. The linear projections of the connective constants for the (a) square, (b) simple cubic and (c) triangular lattices. The last two columns of each row are obtained by use of the estimated exponent γ.

(a)

N	C_N/C_{N-1}	$\mu(N, N-2; 0)$	$\bar{\mu}(N, N-2; 0)$	$\mu_{N,\mathrm{St}}$	$\mu_{\mathrm{St}}(N, N-2; 0)$	$\mu'_N(\gamma = 1.385)$	$\mu'_N(\gamma = 1.333)$
14	2.7951	2.7138	2.7146	2.7982	2.7106	2.7203	2.7302
15	2.7884	2.7213	2.7176	2.7919	2.7149	2.7186	2.7279
16	2.7855	2.7178	2.7195	2.7878	2.7147	2.7200	2.7287
17	2.7800	2.7169	2.7173	2.7828	2.7143	2.7184	2.7266
18	2.7780	2.7180	2.7174	2.7797	2.7151	2.7197	2.7275

(b)		$\mu(N, N-2; 0)$	$\bar{\mu}(N, N-2; 0)$	$\mu_{N,\mathrm{St}}$	$\mu_{\mathrm{St}}(N, N-2; 0)$	$\mu'_N(\gamma = 1.125)$	$\mu'_{N,\mathrm{St}}(\gamma = 1.125)$
7	4.9336	4.8477	4.8465	4.9379	4.8308	4.8484	4.8513
8	4.9240	4.8507	4.8492	4.9268	4.8425	4.8494	4.8510
9	4.9152	4.8509	4.8508	4.9176	4.8464	4.8489	4.8502
10	4.9096	4.8518	4.8513	4.9112	4.8487	4.8499	4.8506
11	4.9033	4.8494	4.8506	4.9049	4.8477	4.8490	4.8497

(c)		$\mu(N, N-1; 0)$	$\mu_{N,\mathrm{St}}$	$\mu_{\mathrm{St}}(N, N-1; 0)$	$\mu'_N(\gamma = 1.385)$	$\mu'_N(\gamma = 1.333)$
8	4.7328	4.4934	4.7408	4.5294	4.5155	4.5436
9	4.7109	4.5362	4.7172	4.5288	4.5177	4.5428
10	4.6936	4.5378	4.6986	4.5311	4.5196	4.5423
11	4.6789	4.5314	4.6831	4.5276	4.5206	4.5414
12	4.6662	4.5274	4.6699	4.5243	4.5212	4.5402

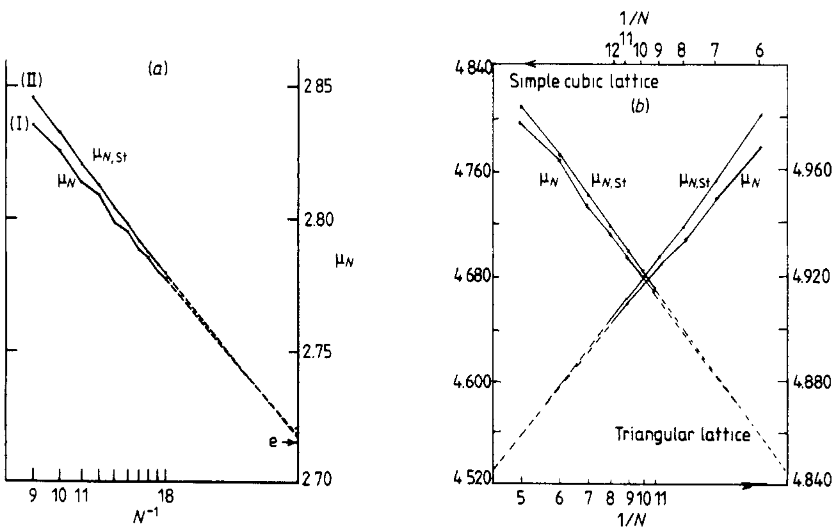

Figure 1. (a) Ratio plot for square lattice. $\mu_N = C_N/C_{N-1}$, $\mu_{N,\mathrm{St}} = \Sigma_{m=1}^{N} C_m / \Sigma_{m=1}^{N-1} C_m$. ($b$) Ratio plot for triangular and simple cubic lattices: μ_N and $\mu_{N,\mathrm{St}}$ are as in (a).

also used to obtain γ'_Ns for comparison. The corresponding figures 2 show that exponents γ'_Ns are very sensitive to the corresponding estimates of μ's. In the square lattice, the value of 2.713 for μ', which had been used by Malakis (1976), is also used for comparison. From table 2, we think that the following exponents may be tempting:

$$\gamma'_s \sim \gamma'_t \sim 1.385, \tag{14}$$

$$\gamma'_{sc} \sim 1.125. \tag{15}$$

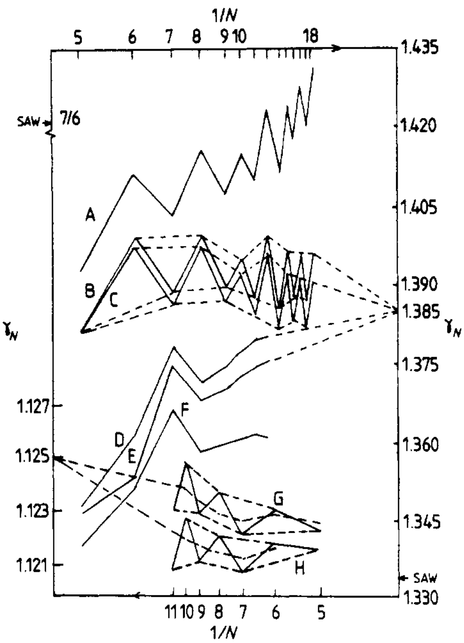

Figure 2. Susceptibility exponents γ_N $(=N(\mu_N/\mu'-1)-1)$ against $1/N$ for the square, triangular and simple cubic lattices. The adopted estimates μ' are: A, $\mu'=2.713$; B, $\mu'=2.7182$; C, $\mu'=2.719$; D, $\mu'=4.523$; E, $\mu'=4.525$; F, $\mu'=4.530$; G, $\mu'=4.849$; H, $\mu'=4.850$.

These exponents are different from those for SAWs. We think the above new exponents may be preferable. Firstly, the exponents in (14) and (15) come from the rather reliable estimates of μ's, as one can see from our figure 1, as well as table 2. Secondly, in Malakis (1976, figure 1) a wide range of α's (the more usual notation for which is $\gamma-1$), 0.32, 0.33 and 0.34, had been used to estimate critical exponents and all of them (on the corresponding μ_N–$(1/N)$ plot) converge to the same point $\mu'=2.713$; while for fitting the SAPW and SAW class, another value of $\mu=2.718\ldots\approx \mathrm{e}$ had to be adopted (see the second columns in Malakis (1976), tables 3 and 4), and if the convergent value 2.713 is adopted (see the third columns in Malakis (1976), tables 3 and 4), then the obtained γ'_Ns always present considerable discrepancies (more than ten per cent) with those for SAWs. One has to acknowledge this unsatisfactory and inconsistent situation. If we assume that (14) and (15) are valid, and reset them into the more accurate expression for μ'_N,

$$\mu'_N = N\mu_N/(N+\gamma'-1) \sim \mu(1+\mathrm{O}(1/N^2)), \tag{16}$$

we get the following refined values (see the last columns of each row in table 2):

$$\mu_s = 2.7182 \pm 0.0015, \tag{17}$$

$$\mu_t = 4.523 \pm 0.003, \tag{18}$$

$$\mu_{sc} = 4.849 \pm 0.001. \tag{19}$$

These values are well consistent with the original ones given by (10) and (11) (figure 3). The self-consistency for 3D is satisfactory (see figure 3(*b*)). Thirdly, for the square and triangular lattices, we nearly obtained the same self-consistent value of $\gamma \sim 1.385$, which is in agreement with the well accepted conclusion that critical exponents only depend on dimensionality. If one insists on the same universality class conclusion, the considerable inconsistency between μ's and γ's reappears (table 2). Lastly, although γ'_Ns are very sensitive to the estimate of μ's and a considerable dispersion appears in our figure 2, one can hardly think how γ'_Ns can approach those for SAWs (see the arrows in figure 2). In other words, our data for susceptibility exponents indicate that not only the values but also the tendencies present a considerable discrepancy with those of SAWs.

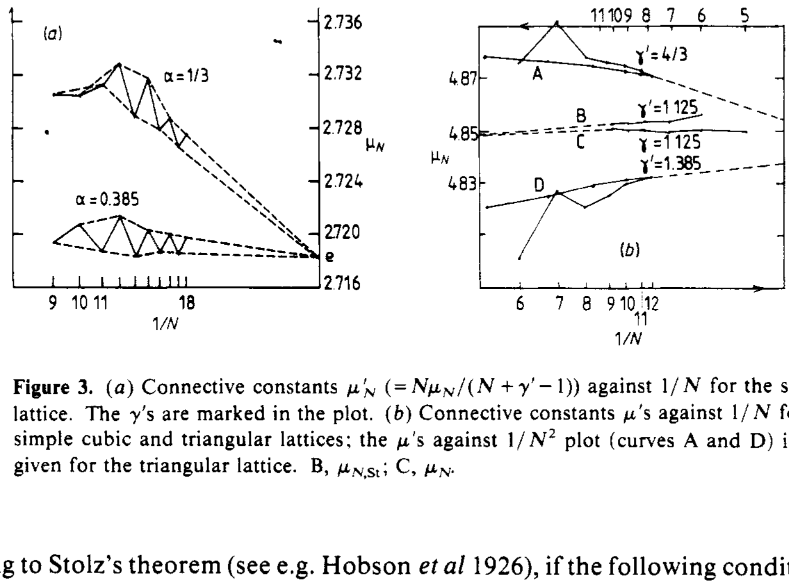

Figure 3. (*a*) Connective constants μ'_N $(= N\mu_N/(N + \gamma' - 1))$ against $1/N$ for the square lattice. The γ's are marked in the plot. (*b*) Connective constants μ's against $1/N$ for the simple cubic and triangular lattices; the μ's against $1/N^2$ plot (curves A and D) is also given for the triangular lattice. B, $\mu_{N,\mathrm{St}}$; C, μ_N.

According to Stolz's theorem (see e.g. Hobson *et al* 1926), if the following conditions are fulfilled for the two series $\{X_N\}$ and $\{Y_N\}$:

$$Y_{N+1} > Y_N; \lim_{N\to\infty} X_N = +\infty \text{ and } \lim_{N\to\infty} Y_N = +\infty,$$

then we have

$$\lim_{N\to\infty} X_N / Y_N = A,$$

if the following limit exists: $\lim_{N\to\infty} (X_N - X_{N-1})/(Y_N - Y_{N-1}) = A$.

We define series $\{X_N\}$ and $\{Y_N\}$ by

$$X_N = \sum_{m=0}^{N} C_m, \qquad Y_N = \sum_{m=0}^{N-1} C_m, \qquad C_0 = 1,$$

Table 3. The susceptibility exponents for the (*a*) square, (*b*) simple cubic and (*c*) triangular lattices. The connective constants used appear in brackets.

(*a*)

N	$\gamma_N(2.719)$	$\gamma_N(2.718)$	$\gamma_N(2.713)$	$\gamma_{N,\mathrm{St}}(2.719)$	$\gamma_{N,\mathrm{St}}(\mathrm{e})$	$\gamma_{\mathrm{St}}(N, N-2, \varepsilon=0; \mu=\mathrm{e})$
14	1.3919	1.3872	1.4397	1.4078	1.4121	1.3792
15	1.3829	1.3886	1.4363	1.4022	1.4068	1.3840
16	1.3910	1.3970	1.4409	1.4047	1.4095	1.3913
17	1.3814	1.3877	1.4373	1.3988	1.4039	1.3867
18	1.3903	1.3970	1.4425	1.4018	1.4072	1.3855

(*b*)	$\gamma_N(4.850)$	$\gamma_N(4.849)$	$\gamma_N(4.848)$	$\gamma_{N,\mathrm{St}}(4.850)$	$\gamma_{N,\mathrm{St}}(4.849)$	$\gamma_{N,\mathrm{St}}(4.848)$
7	1.1207	1.1222	1.1236	1.1269	1.1284	1.1298
8	1.1221	1.1238	1.1254	1.1267	1.1284	1.1301
9	1.1211	1.1229	1.1248	1.1254	1.1273	1.1292
10	1.1228	1.1249	1.1270	1.1262	1.1283	1.1304
11	1.1208	1.1231	1.1254	1.1245	1.1268	1.1291

(*c*)	$\gamma_N(4.523)$	$\gamma_N(4.525)$	$\gamma_N(4.527)$	$\gamma_{N,\mathrm{St}}(4.523)$	$\gamma_{N,\mathrm{St}}(4.524)$
8	1.3710	1.3673	1.3636	1.3852	1.3833
9	1.3739	1.3698	1.3656	1.3864	1.3844
10	1.3772	1.3726	1.3680	1.3882	1.3859
11	1.3790	1.3740	1.3690	1.3893	1.3867
12	1.3800	1.3745	1.3691	1.3895	1.3868

(for which the conditions of Stolz's theorem are fulfilled obviously); then we have

$$\lim_{N\to\infty}\left(\sum_{m=0}^{N} C_m \Big/ \sum_{m=0}^{N-1} C_m\right) = \mu,$$

if the following limit exists: $\lim_{N\to\infty} C_N/C_{N-1} \equiv \lim_{N\to\infty} \mu_N = \mu$. Not only has one the equality of the two limits for the series $\{C_N/C_{N-1}\}$ and $\{X_N/Y_N\}$, but we can also show that they have the same asymptotic behaviour by the following few steps:

$$(C_0+C_N)/2 + \sum_{l=1}^{N-1} C_l \sim \int_0^N C(x)\,\mathrm{d}x \sim \int_K^N x^{\gamma-1}\mu^x\,\mathrm{d}x,$$

in which the trapezoidal rule and the asymptotic form (5) for C_N have been used, and K is any finite constant.

$$\int_K^N x^{\gamma-1}\mu^x\,\mathrm{d}x = \left(x^{\gamma-1}\mu^x\Big|_K^N - \int_K^N (\gamma-1)x^{\gamma-2}\mu^x\,\mathrm{d}x\right)\Big/\ln\mu \sim N^{\gamma-1}\mu^N/\ln\mu,$$

where we have used $\gamma>1$, $\mu>0$ and $N\gg 1$; thus we have the following asymptotic formula:

$$\frac{\sum_{m=0}^{N} C_m}{\sum_{m=0}^{N-1} C_m} \sim \frac{\int_K^{N+1} C(x)\,\mathrm{d}x - C_{N+1}/2}{\int_K^N C(X)\,\mathrm{d}x - C_N} \sim \frac{(N+1)^{\gamma-1}\mu^{N+1}/\ln\mu - C_{N+1}/2}{N^{\gamma-1}\mu^N/\ln\mu - C_N/2}$$

$$\sim \frac{\mu^{N+1}(N+1)^{\gamma-1}}{\mu^N N^{\gamma-1}} \sim \mu[1+(\gamma-1)/N],$$

i.e.

$$\sum_{m=1}^{N} C_m \Big/ \sum_{m=1}^{N-1} C_m \sim \mu[1+(\gamma-1)/N] \sim C_N/C_{N-1} \equiv \mu_N. \tag{20}$$

An alternative way to argue the validity of (20) is as follows: $\Sigma_{m=1}^{N} C_m/\Sigma_{m=1}^{N-1} C_m \sim \Sigma_{m=1}^{N} C_m/C_N \times C_{N-1}/\Sigma_{m=1}^{N-1} C_m \times C_N/C_{N-1} \sim [1+\delta(N)-\delta(N-1)] \times C_N/C_{N-1}$, in which Stolz's theorem has been applied to the series $\{\Sigma_{m=1}^{N} C_m/C_N\}$, and we get $\Sigma_{m=1}^{N} C_m/C_N \sim [\mu/(\mu-1)][1+\delta(N)]$, and $\delta(N) \to_{N\to\infty} 0$. For the usual asymptotic behaviours of $\delta(N)$, one has $N[\delta(N)-\delta(N-1)] \to_{N\to\infty} 0$ and thus the asymptotic form (20) is obtained.

Using (20), the results for the square, triangular and simple cubic lattices are shown in figures 1 and 3. Hereafter the subscript St is used to denote the approach by Stolz's theorem. When comparing with the original direct ratio $\mu_N = C_N/C_{N-1}$ one can find that the results for two different average methods are well consistent, and the approach by Stolz's theorem appears to be smoother and better than the other one.

3. Analysis of mean square sizes

The purpose of this section is to utilise the results obtained by the method of exact enumeration to analyse the mean square end-to-end distance. In table 1 we give the integer $\rho_N C_N$ for the square and simple cubic lattices.

According to (6), except for the constant coefficient there is only one unknown parameter, the correlation-length exponent ν. Thus the situation in this section is simpler than before. As in § 2, two different average approaches are used here. We define two series $\{X_m\}$ and $\{Y_m\}$ as follows: $X_m = \Sigma_{\{w_m\}} \rho_{w_m}$, $Y_m = \Sigma_{\{w_m\}} 1$, where $\{w_m\}$ and ρ_{w_m} are the distinguished m-step paths and the square end-to-end distance respectively. We have

$$\rho_N = X_N/Y_N \sim AN^{2\nu} \tag{21}$$

and the corresponding

$$\rho_{N,\mathrm{St}} = \sum_{m=1}^{N} X_m \Big/ \sum_{m=1}^{N} Y_m. \tag{22}$$

Since $\{\rho_N\}$ is a divergent series, one has to prove the legality of using Stolz's theorem first. By application of Stolz's theorem to the ratios of ρ_Ns and $\rho_{N,\mathrm{St}}$s

$$\rho_N/\rho_{N,\mathrm{St}} \sim AN^{2\nu} \sum_{m=1}^{N} Y_m \Big/ \sum_{m=1}^{N} X_m \sim A\left[N^{2\nu} \sum_{m=1}^{N} Y_m - N^{2\nu}\left(1-\frac{1}{N}\right)^{2\nu} \sum_{m=1}^{N-1} Y_m\right] \Big/ X_N.$$

Since $1-2\nu|N < (1-1/N)^{2\nu} < 1$ $(N \gg 1, N>0)$, we have $1 < \rho_N/\rho_{N,\mathrm{St}} < 1+2\nu$, in other words, the dominant terms of the two divergent series $\{\rho_N\}$ and $\{\rho_{N,\mathrm{St}}\}$ have the same degree of divergence $\sim N^{2\nu}$. The two kinds of ν_Ns are defined as follows:

$$\nu_N = \tfrac{1}{2}N(\rho_{N+1}/\rho_N - 1) \sim \nu[1+\mathrm{O}(1/N)] \tag{23}$$

and

$$\nu_{N,\mathrm{St}} = (N/2i)(\rho_{N+i,\mathrm{St}}/\rho_{N,\mathrm{St}} - 1) \sim \nu[1+\mathrm{O}(1/N)]. \tag{24}$$

In table 4 we list ν_Ns and their linear projections. Also the property of even–odd oscillation for loose packed lattices has been considered. Our data show again that

Table 4. Correlation-length exponents for the (*a*) square and (*b*) simple cubic lattices.

(*a*)

N	$2\nu_N$	$2\nu(N, N-2; 0)$	$2\nu_{N,\mathrm{St}}$	$2\nu_{\mathrm{St}}(N, N-2; 0)$
12	1.410 06	1.482 10	1.462 88	1.484 98
13	1.404 61	1.482 25	1.460 28	1.492 40
14	1.418 05	1.466 03	1.465 87	1.483 81
15	1.414 09	1.475 73	1.463 61	1.485 26
16	1.427 45	1.493 20	1.469 68	1.496 35
17	1.422 12	1.482 35	1.467 09	1.493 19
(*b*)				
6	1.136 54	1.122 35	1.184 10	1.128 42
7	1.133 40	1.118 87	1.174 74	1.122 54
8	1.132 65	1.120 99	1.168 63	1.122 01
9	1.130 21	1.119 04	1.162 51	1.120 12
10	1.131 31	1.125 95	1.159 90	1.125 01

the results obtained by (24) appear to be smoother and better than those obtained by (23). From table 4 we get

$$2\nu_{2\mathrm{D}} \approx 1.490, \tag{25}$$

$$2\nu_{3\mathrm{D}} \approx 1.125 = \tfrac{9}{8}, \tag{26}$$

which are well consistent with the extrapolated values from some nearly straight lines in figure 4. The correlation-length exponent is very close to that of SAWs for 2D, while for 3D it is different from the SAW one, and is close to our PSRG result (Li *et al* 1984).

4. Conclusion

A simple, peculiar and seldom studied excluded volume effect model, the SAPW, is studied by the series expansion method. Since there is a very different global excluded-volume effect from that of SAWs, one may expect the possibility of emergence of a new universality class for SAPWs. For the interest and worth of this problem, we re-examine the previous strong suggestion that trails and SAWs belong to the same universality class. After checking the previous results we found that one needs to revise the exact enumeration of paths on the square lattice. An exact enumeration for the square, triangular and simple cubic lattices is done by computer up to 18, 12, and 11 steps respectively. In addition to the traditional average method in series expansion, another approach based on Stolz's theorem is used and it appears to be smoother and better than the former one. Satisfactory straight lines are obtained in our plot of connective constant against $1/N$ for three kinds of lattices; thus we think that the following connective constants are reliable: $\mu_s = 2.7182 \pm 0.0015$, $\mu_t = 4.523 \pm 0.003$ and $\mu_{sc} = 4.849 \pm 0.001$. Considerable dispersions are present when one uses the above connective constants to deduce susceptibility exponents. This also happens to SAWs. Since $\ln C_N = N \ln \mu + (\gamma - 1) \ln N \sim N \ln \mu$, to deduce the small term from an estimate of the dominant term, one has to run risks. Our results for 2D lattices indicate the possibility of a different class for SAPWs. If one insists on the same class conclusion,

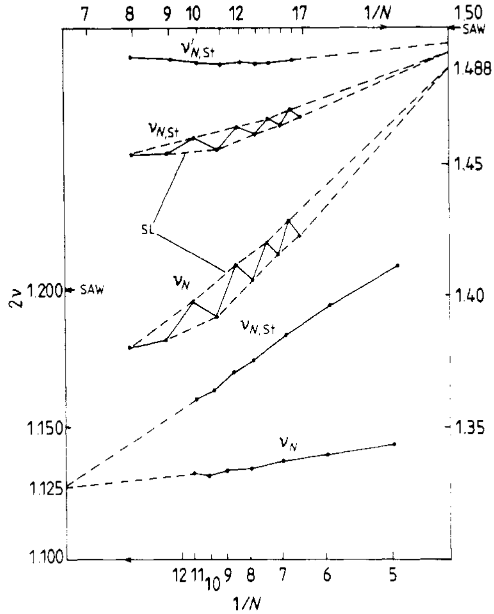

Figure 4. Correlation-length exponent ν_N as a function of $1/N$ for the square and simple cubic lattices. The subscript St denotes the average approach by use of Stolz's theorem. After initial irregularities the 2D ν_N apparently tends to the same value as for SAWs. The 3D ν_N of two different average approaches apparently tend to the same value $\frac{9}{16}$, in contrast with the SAW Flory value $\frac{3}{5}$.

then one has to tolerate a considerable inconsistency between connective constant and susceptibility exponent, which happens at least for the up-to-date limited number of exact enumerations; to keep consistency, a considerable difference exists between the values given here and that of the SAW. Our results for 2D show that the correlation-length exponent ν_N unambiguously approaches or even equals that of SAWs. Our results for 3D present a satisfactory linear relation between ν_N and $1/N$, thus $\nu \sim \frac{9}{16}$ seems more tempting than $\frac{3}{5}$ of the SAW.

We should mention that only a conventional series expansion result is presented in this paper, and no confluent corrections to the dominant asymptotic behaviour have been considered above. Some very recently published papers (e.g. Djordjevic *et al* 1983) have pointed out the importance of confluent corrections when estimating the exponent ν. After doing some confluent corrections, our result is $\nu_{3D} \sim 0.5729 \pm 0.0002$, which is extremely close to our direct PSRG result, $\nu_{3D}(b=2) = 0.5728$ (Li *et al* 1984). This near coincidence had happened to SAWs too, in which the $\nu_{3D}(b=2) = 0.5875$ (Family 1981) is just the same as that of Djordjevic *et al* (1983). Our result of 0.5729 still has considerable deviation from 0.5875 of SAWs, even though the confluent effects have been taken into account for series expansion in both cases.

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